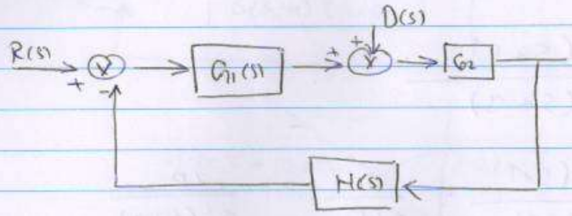
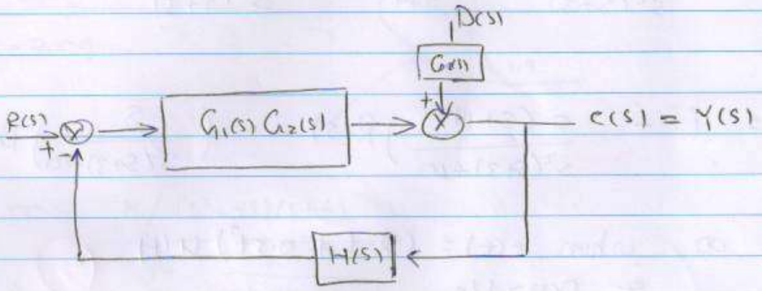
Exam Solution

Q#1:





$E(s) = R(s) - Y(s)$

$E(s) = R(s) - C(s)$

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$[R(s) - H(s)\,C(s)]\,G_1(s)\,G_2(s) + D(s)\,G_2(s) = C(s)$

$G_1(s)\,G_2(s)\,R(s) - H(s)\,G_1(s)\,G_2(s)\,C(s) + D(s)\,G_2(s) = C(s)$

$G_1(s)\,G_2(s)\,R(s) + D(s)\,G_2(s) = (1 + H(s)\,G_1(s)\,G_2(s))\,C(s)$

$$C(s) = \left(\frac{G_1 G_2}{1 + H G_1 G_2}\right) R(s) + \frac{G_2}{1 + H G_1 G_2} D(s)$$

$$E(s) = \left(1 - \frac{G_1 G_2}{1 + H G_1 G_2}\right) R(s) - \left(\frac{G_2}{1 + H G_1 G_2}\right) D(s)$$

$$G_1(s)G_2(s) = \frac{5(s+1)}{s} * \frac{1}{s(s+3)}$$

$$= \frac{5(s+1)}{s^2(s+3)}$$

$$H G_1 G_2 = \frac{5(s+1)}{s^2(s+3)} \cdot \frac{2}{s+1} = \frac{10}{s^2(s+3)}$$

$$E(s) = \left(1 - \frac{5(s+1)}{s^2(s+3)+10}\right)R(s) - \left(\frac{5}{s^2(s+3)+10}\right)D(s)$$

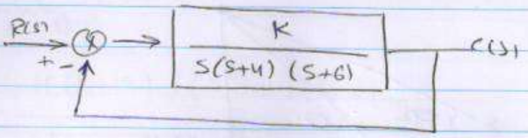
$e(\infty) = \infty$ when $r(t) = (2 - t + 0.5t^2)\,u(t)$ & $D(s) = 1/s$ (5)

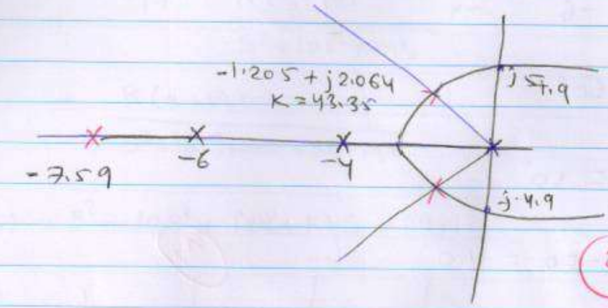
when $r(t) = 0$ & $D(s) = 1/s$

$$E(s) = -\frac{1}{s^2(s+3)+10}$$

$$e(\infty) = \lim_{s \to 0} s\,E(s) = 0$$ (5)

Q#2:





5 ×2

$$TF = \frac{K/(s^2+4s)(s+6)}{1+\frac{K}{(s^2+4s)(s+6)}}$$

$$P(s) = s^3 + 10s^2 + 24s + K$$

s^3	1	24	
s^2	10	K	
s^1	$-\frac{(K-240)}{10}$	0	0
s^0	$+K$	0	0

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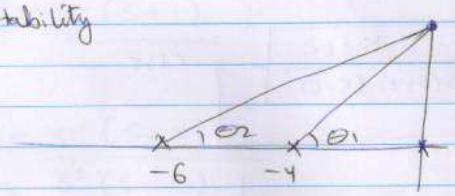
$K - 240 = 0 \Rightarrow K = 240 \Rightarrow K < 240$ for stability

$10s^2 + 240 = 0 \Rightarrow s = \pm j4.8$

To improve stability



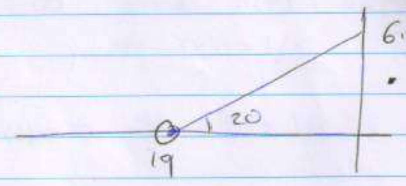
$\theta_1 = \tan^{-1}\left(\frac{6.9}{4}\right) = 60°$

$\theta_2 = \tan^{-1}\left(\frac{6.9}{a}\right) = 50°$

$-\theta_1 - \theta_2 = -60 - 50 = -110$

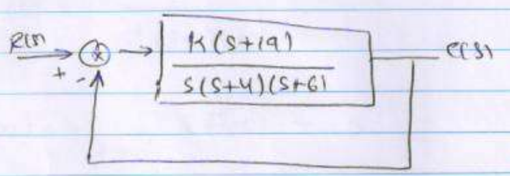
3

$\Rightarrow \theta_z = 90$ since $\theta_z - 110 = 180$



$\tan(20) = \frac{6.9}{\alpha}$

$\boxed{\alpha = 19}$



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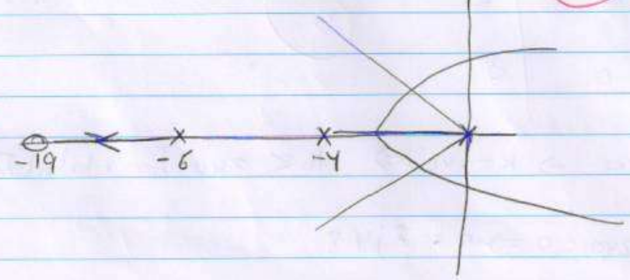


For stability

$$Tf = \frac{K(s+19) / (s^2+4s)(s+6)}{1 + \frac{K(s+19)}{s^3+10s^2+24s}}$$

$$= \frac{K(s+19)}{s^3+10s^2+24s+Ks+19K}$$

$$p(s) = s^3 + 10s^2 + (24+K)s + 19K$$

s^3	1	24+K	0
s^2	10	19K	0
s^1	$-\left(\frac{9k-240}{10}\right)$	0	0
s^0	19K	0	0

(5)

For stability $-9k+240 > 0$

$$\boxed{k < 26.7}$$

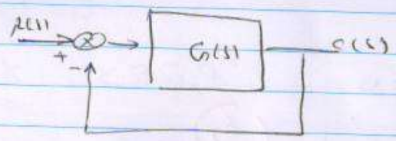
$$10s^2 + (19)(26.7) = 0$$

$$\boxed{s = \pm j6.9}$$

The new system, settling time is shorter but with the same percent overshoot.

(2)

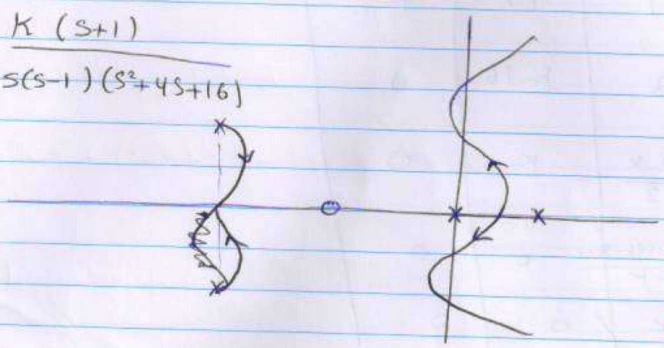
Q#3:



$$G(s) = \frac{K(s+1)}{s(s-1)(s^2+4s+16)}$$

①

(6)



② Angle of asymptotes $= \frac{180(2k+1)}{4-1} = 60, -60, 180$

(5) $\sigma_a = -\frac{(0-1+2+j2\sqrt{3}+2-j2\sqrt{3})}{4-1} = -\frac{2}{3}$

③ breakaway & break-in points

(3) $1 + \frac{K(s+1)}{s(s-1)(s^2+4s+16)} = 0 \Rightarrow K = -\frac{(s)(s-1)(s^2+4s+16)}{s+1}$

$$\frac{\partial k}{\partial s} = -\frac{3s^4+10s^3+21s^2+24s-16}{(s+1)^2}$$

$3s^2+10s^3+21s^2+24s-16 = 3(s+0.76+j2.16)(s+0.76-j2.16)$
$(s+2.26)(s-0.45)$

breakaway point at 0.45

break in point at -2.26

(4) By using Routh table

$s^4 + 3s^3 + 12s^2 + (K-16)s + K = 0$

(3)

s^4	1	12	K
s^3	3	K-16	0
s^2	$\frac{52-K}{3}$	K	0
s^1	$\frac{-K^2+59K-832}{52-K}$	0	0
s^0	K	0	0

$\frac{52-K}{3} s^2 + K = 0$

$\frac{-K^2+59K-832}{52-K} = 0 \Rightarrow K = 35.7$ & $K = 23.3$

$\Rightarrow$

$s = \pm j\,2.56$ for $K = 35.7$

$s = \pm j\,1.56$ for $K = 23.3$

(5) The angle of departure

$\theta = 180 - 120 - 130.5 - 90 + 106$

$= -54.5^\circ$

(5)

(6) For stability

(3)

$23.3 < k < 35.7$°

otherewise the system unstable.